Published by Institute of Physics Publishing for SISSA/ISAS
Received: *March 19, 2003*
Revised: *March 25, 2003*
Accepted: *March 31, 2003*

Long-distance dimension-eight operators in B_K

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Abstract: Besides their appearance at short distances $\gtrsim 1/M_W$, local dimension-eight operators also contribute to kaon matrix elements at long distances of order $\gtrsim 1/\mu_{\rm ope}$, where $\mu_{\rm ope}$ is the scale controlling the Operator Product Expansion in pure QCD, without weak interactions. This comes about in the matching condition between the effective quark lagrangian and the Chiral lagrangian of mesons. Working in dimensional regularization and in a framework where these effects can be systematically studied, we calculate the correction from these long-distance dimension-eight operators to the renormalization group invariant $\hat{B}_K$ factor of $K^0 - \bar{K}^0$ mixing, to next-to-leading order in the $1/N_c$ expansion and in the chiral limit. The correction is controlled by the matrix element $\langle 0|\bar{s}_L \tilde{G}_{\mu\nu}\gamma^\mu d_L|K^0\rangle$, is small, and lowers $\hat{B}_K$.

Keywords: 1/N Expansion, Standard Model, Kaon Physics, Chiral Lagrangians.

 http://jhep.sissa.it/archive/papers/jhep032003060/jhep032003060.pdf

Contents

1. Introduction

In order to study the physics at the scale of the K meson one can integrate out the fields describing all the other heavier particles. In the Standard Model, this integration generates a local $\Delta S = 2$ operator of the form [1]

$$\mathcal{H}^{\rm eff}_{\Delta S=2} = \frac{G_F^2 m_W^2}{4\pi^2}[\lambda_c^2 F_1 + \lambda_t^2 F_2 + 2\lambda_c\lambda_t F_3]\ C_{\Delta S=2}(\mu)\mathcal{Q}_{\Delta S=2}(x)\,, \tag{1.1}$$

where $C_{\Delta S=2}$ is the Wilson coefficient given (in the large-N_c limit) by

$$C_{\Delta S=2}(\mu) = \left[1+\frac{\alpha_s(\mu)}{\pi}\left(\frac{1433}{1936}+\frac{\kappa}{8}\right)\right]\left(\frac{1}{\alpha_s(\mu)}\right)^{\frac{9}{11N_c}}\ . \tag{1.2}$$

The Wilson coefficient encodes the ultraviolet physics which has been integrated out and is computed in perturbation theory. Consequently, it depends on the renormalization scale (μ) and scheme (κ). In eq. (1.2) $\kappa = 0$ or -4 depending on whether one has used naive dimensional regularization or the 't Hooft-Veltman scheme (respectively) in the calculation. The operator $\mathcal{Q}_{\Delta S=2}(x)$ in eq. (1.1) reads

$$\mathcal{Q}_{\Delta S=2}(x) = \bar{s}_L(x)\gamma_\mu d_L(x)\bar{s}_L(x)\gamma^\mu d_L(x)\,, \qquad \psi_{R,L} \equiv \frac{1\pm\gamma_5}{2}\,\psi\,. \tag{1.3}$$

The F_i are functions of the heavy particles which have been integrated out and their expression can be found in [1].

The invariant bag factor $\hat{B}_K$ is then defined in terms of this effective weak hamiltonian as

$$\langle \bar{K}^0|C_{\Delta S=2}(\mu)\mathcal{Q}_{\Delta S=2}(0)|K^0\rangle \equiv \frac{4}{3}F_K^2 M_K^2 \hat{B}_K \tag{1.4}$$

and, by construction, it is renormalization scheme and scale independent.

Because of confinement and chiral symmetry breaking, there is another effective description which involves directly the lowest-lying mesons as degrees of freedom in a chiral lagrangian organized in powers of momenta and meson masses. In the case of $\Delta S = 2$ interactions, the lowest-order operator in this chiral lagrangian is given by

$$\mathcal{L}^{\chi}_{\Delta S=2} = \frac{F_0^4}{4} g_{\Delta S=2}(\mu)\, \mathrm{Tr}\left(\lambda_{32} U^{\dagger} r^{\mu} U \lambda_{32} U^{\dagger} r_{\mu} U\right) , \tag{1.5}$$

where $(\lambda_{32})_{ij} = \delta_{i3}\delta_{2j}$ is a (spurion) matrix in flavor space, $F_0 \simeq 0.087\,\mathrm{GeV}$ is the pion decay constant in the chiral limit and U is a 3×3 unitary matrix collecting the Goldstone boson excitations and transforming as $U \to RUL^{\dagger}$ under a combined unitary flavor rotation (R, L) of $\mathrm{SU}(3)_R \times \mathrm{SU}(3)_L$. In (1.5) $g_{\Delta S=2}(\mu)$ is a low energy coupling to be determined, as usual, by a matching condition to the lagrangian (1.1). This condition is a crucial ingredient in the construction of the chiral lagrangian (1.5) and secures that the ultraviolet behavior of (1.5) be the same as that of (1.3) so that physics is independent of conventions such as the renormalization scale μ, the renormalization scheme (i.e. NDR vs HV), etc.

Although matching conditions are commonplace in the construction of effective lagrangians such as (1.1) in a perturbative context, it is only recently that their crucial role for the construction of the lagrangian (1.5) has been appreciated also in the nonperturbative context of analytic calculations of electroweak matrix elements of light mesons [3].

In the particular case of B_K and $g_{\Delta S=2}(\mu)$, a convenient Green's function to do the matching with is [2]

$$\mathcal{W}^{\mathrm{LRLR}}_{\mu\alpha\nu\beta}(q,l) = \lim_{l\to 0} i^3 \int d^4x\, d^4y\, d^4z\, e^{iqx} e^{il(y-z)} \langle 0| T\{L^{\bar{s}d}_{\mu}(x) R^{\bar{d}s}_{\alpha}(y) L^{\bar{s}d}_{\nu}(0) R^{\bar{d}s}_{\beta}(z)\}|0\rangle , \tag{1.6}$$

where

$$L^{\bar{s}d}_{\mu}(x) = \bar{s}(x)\gamma_{\mu}\frac{1-\gamma_5}{2} d(x) \qquad R^{\bar{d}s}_{\mu}(x) = \bar{d}(x)\gamma_{\mu}\frac{1+\gamma_5}{2} s(x) \tag{1.7}$$

are QCD chiral currents. It is a general property that chiral coupling constants such as $g_{\Delta S=2}$ in eq. (1.5) can always be defined by means of QCD Green's functions — in the forward limit — which are chiral order parameters, and hence vanish to all orders in perturbation theory in the chiral limit.

Performing a standard analysis of Ward identities leads to the following relation

$$\int d\Omega_q\, g^{\mu\nu}\, \mathcal{W}^{\mathrm{LRLR}}_{\mu\alpha\nu\beta}(q,l)_{\mathrm{unfactorized}} = \left(\frac{l^{\alpha}l^{\beta}}{l^2} - g^{\alpha\beta}\right) \mathcal{W}^{\mathrm{LRLR}}(Q^2) , \qquad Q^2 \equiv -q^2 , \tag{1.8}$$

where $\int d\Omega_q$ stands for the average over the momentum q in D dimensions, namely

$$\int d\Omega_q\, q_{\mu}q_{\nu}\, f(q^2) = \frac{q^2 g_{\mu\nu}}{D}\, f(q^2) , \tag{1.9}$$

for a given function $f(q^2)$. Using then the definitions

$$z = \frac{Q^2}{\mu^2_{\mathrm{had}}} , \qquad \mathcal{W}^{\mathrm{LRLR}}(z) = \frac{F_0^2}{z\mu^2_{\mathrm{had}}} W(z) , \tag{1.10}$$

one can express $g_{\Delta S=2}$ in eq. (1.5) as [2]

$$g_{\Delta S=2}(\mu,\epsilon) = 1 - \frac{\mu^2_{\mathrm{had}}}{32\pi^2 F_0^2} \left(\frac{4\pi\mu^2}{\mu^2_{\mathrm{had}}}\right)^{\epsilon/2} \frac{1}{\Gamma(2-\epsilon/2)} \int_0^{\infty} dz\, z^{-\epsilon/2} W(z) . \tag{1.11}$$

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This equation expresses the matching condition between the chiral meson lagrangian (1.5) and the quark lagrangian (1.1). We emphasize that the scale $\mu^2_{\rm had}$ in eq. (1.11) is used solely for a rescaling of the momentum Q^2 and, in principle, is totally arbitrary. In practice, however, there is always a residual dependence on $\mu^2_{\rm had}$ in the final result since calculations are done to a finite order in, e.g., α_s. Therefore this scale $\mu^2_{\rm had}$ has to be numerically large enough so as to make meaningful the truncated series in α_s. Physically, one can think of $\mu^2_{\rm had}$ as the scale at which resonances are integrated out and the lagrangian (1.5) sets in.

From eq. (1.11) it is clear that $g_{\Delta S=2}(\mu)$ requires the knowledge of the Green's function $W(z)$ over the full range of momenta and regretfully, we hasten to say, this information is not available. What is available, however, is its high- and low-z behavior since they are given by the Operator Product and Chiral expansions, respectively. Also, it is known that the analytic structure of $W(z)$ simplifies notably in the large-N_c limit becoming a meromorphic function, i.e. consisting of only poles, with no cuts. However, lacking the solution to QCD at large N_c, the problem is still too difficult to tackle even in this limit: the masses and residues of the infinite number of resonances contributing to $W(z)$ are not known. Obviously, a further approximation is necessary and this has been termed "The Hadronic Approximation to large-N_c QCD" (HA).

In order to motivate this approximation, it is important to realize that it is not the point-wise behavior of $W(z)$ that is necessary, but only its integral. Moreover, $W(z)$ is defined fully in the euclidean regime. Therefore, it is reasonable to expect that a smooth interpolation between the low-z and the high-z regimes of $W(z)$ may do a good job in approximating the integral. Consequently we shall construct the HA as a rational approximation[1] to large-N_c QCD by keeping only a *finite* number of zero-width resonances whose masses and residues are obtained from matching to the first few coefficients of the Operator Product and Chiral expansions. In the case of the coupling $g_{\Delta S=2}$ in eq. (1.11), these expansions correspond to the high- and low-z behavior of the function $W(z)$, respectively.

The rational approximant so constructed, $W_{HA}(z)$, is therefore an interpolating function between the high-z and low-z tails of $W(z)$ which it matches by construction. The more terms one knows in the Operator Product and Chiral expansions, the more resonances one can determine from the matching to construct the interpolator $W_{HA}(z)$. Therefore the approximation is, in principle, improvable. In practice, however, often only one or two terms in each expansion are known, so that one actually has to borrow the masses of the resonances from the Particle Data tables and leave only the residues to be determined by these matching conditions.[2] There is a minimal requirement, though, which is that $W_{HA}(z)$ *has* to reproduce at least the leading non-vanishing term in the OPE. This is indispensable for accomplishing cancelation of scale and renormalization scheme dependence with Wilson coefficients, such as $C_{\Delta S=2}(\mu)$. This was done in the calculation of $\hat{B}_K$ in ref. [2]. Several other observables have also been computed within this framework [4, 5].

We may now take the example of the matching condition (1.11) to discuss the appearance of local dimension-eight operators at relatively long distances. Indeed, the large-z fall-

[1] i.e. it can be written as a ratio of two polynomials.

[2] If enough conditions were known one could also determine the masses, of course. For a model in which this is done, see ref. [7]

off of the function $W(z)$ in this matching condition between the chiral meson lagrangian and the quark lagrangian is given by the OPE of the Green's function (1.6). Dimension-eight operators (and higher) contribute to this OPE and thus they appear in kaon matrix elements at rather long distances, i.e. *without* short-distance suppression factors such as $1/M_W^2$. Notice that, despite being local, these dimension-eight operators do not appear in the quark effective lagrangian.[3]

The appearance of these operators has been correctly emphasized in [6], particularly in approaches with the use of a momentum cutoff. However, we remark that this contribution from dimension-eight operators is not caused by the cutoff and, in fact, is also true even if one uses a purely dimensional regularization, as in eq. (1.11). Clearly, a physical effect can not depend on the particular regularization employed. What happens in approaches using a momentum cutoff Λ (and does not happen in dimensional regularization) is that, by pure dimensional analysis, local dimension-eight operators can also appear at distances $\gtrsim 1/\Lambda$ suppressed by $1/\Lambda^2$.[4] Moreover, the authors of [6] have cautioned that these long-distance contributions from dimension-eight operators may give potentially large contributions to weak matrix elements.

In our large-N_c expansion approach, based on dimensional regularization, we shall see that dimension-eight operators produce contributions which can be roughly characterized as of $\mathcal{O}(\alpha_s\ \delta^2/\mu_{\mathrm{ope}}^2)$, where δ^2 is a Goldstone-to-vacuum matrix element and μ_{ope} is the scale controlling the Operator Product Expansion of correlators in pure QCD, i.e. *without* weak interactions. To be a bit more precise about this scale, we may define it in general as the momentum scale above which the OPE in pure QCD starts making sense, even if this is only in the limited sense of an asymptotic expansion. Phenomenologically this scale μ_{ope} may in general be somewhat dependent on the quantum numbers involved, but it is usually of the order of a resonance mass (i.e. $\sim 1\,\mathrm{GeV}$).

Besides the long-distance contributions alluded to above, there are other short-distance contributions which are also due to local dimension-eight operators but which appear after the integration of the W boson or, in general, of any heavy field. These contributions, unlike the former, *are* encoded in the quark effective lagrangian. Indeed, when writing the effective hamiltonian of eq. (1.1) one is neglecting dimension-eight operators at short distances which appear below the charm mass, m_c, after integration of this quark. Consequently, these dimension-eight operators appear suppressed by the short distance scale $1/m_c^2$. The size of this $1/m_c^2$ effect is also an interesting problem in itself but, since it can be dealt with by ordinary methods, it will not be the subject of the present work.[5] We plan to come back to it in the future.

In this work we would like to quantitatively investigate the impact of the $1/\mu_{\mathrm{ope}}^2$-suppressed dimension-eight operators in approaches based on dimensional regularization,

[3]In a way, they correspond to having integrated all the meson resonances out of the lagrangian. This is why only Goldstone degrees of freedom appear in (1.5). This integration can not be encoded in a quark effective operator in the lagrangian because this would give rise to a double counting problem: this quark effective operator would generate both Goldstone mesons *and* the resonances which one thought one had integrated out!

[4]In the lattice jargon they are known as $\mathcal{O}(a)$ effects. For us, $a \sim 1/\Lambda$.

[5]Some $1/m_c$ effects have been already considered, for example, in ref. [8].

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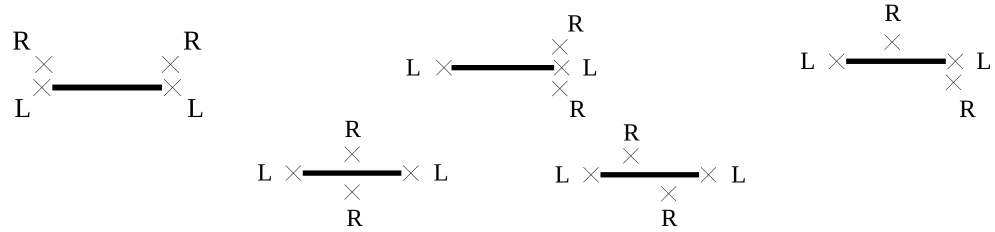

Figure 1: Resonance diagrams contributing to $W(z)$.

such as the HA. If dimension-eight operators are to give an important contribution in this case, they should correct significantly the lowest order term in the OPE at large Q^2, i.e. the high-z tail of functions such as $W(z)$ in the determination of $g_{\Delta S=2}(\mu)$ in eq. (1.11). In the following we present a new evaluation of $\hat{B}_K$ using the Hadronic Approximation to large-N_c QCD with inclusion of dimension-eight operators in the OPE. We now turn to the details of this analysis.

After chiral corrections, the result in the $N_c \to \infty$ limit stems from the full factorization of the operator (1.3) into two left-handed currents with the result

$$B_K(N_c \to \infty) = \frac{3}{4} \qquad \text{and} \qquad C_{\Delta S=2}(\mu)(N_c \to \infty) = 1\,. \tag{1.12}$$

Including the next-to-leading order in the $1/N_c$ expansion one obtains

$$\hat{B}_K = \frac{3}{4}\, C_{\Delta S=2}(\mu)\; g_{\Delta S=2}(\mu)\,, \tag{1.13}$$

where $g_{\Delta S=2}(\mu)$ is given by eqs. (1.11) and $C_{\Delta S=2}(\mu)$ is given by eq. (1.2). We emphasize that the function $\mathcal{W}^{\mathrm{LRLR}}(Q^2)$ in eq. (1.8) is of $\mathcal{O}(N_c)$ and, therefore, it is given by tree-level diagrams with infinitely narrow resonances. After the redefinition (1.10), the function $W(z)$ is of $\mathcal{O}(N_c^0)$ and, recalling that $F_0^2 \sim \mathcal{O}(N_c)$, $g_{\Delta S=2}(\mu)$ is indeed of the form $\sim 1-\mathcal{O}(1/N_c)$, as one would expect.

In a combined large-N_c and quark mass (m_q) expansion, the result in eq. (1.13) is only corrected by terms of $\mathcal{O}(m_q/N_c)$. Furthermore, it is also interesting to emphasize that if F_0 is used instead of F_K in the definition of $\hat{B}_K$ in eq. (1.4), then the above expression (1.13) is also what is being computed on the lattice as the $\hat{B}_K$ parameter extrapolated to the chiral limit. This fact is very convenient as it allows for a comparison between the present large-N_c method and the lattice results in this limit.

Figure 1 shows the different topologies of the resonance diagrams contributing to $W(z)$ in eqs. (1.6)–(1.11) in the large-N_c limit. Use of the Mittag-Leffler theorem for meromorphic functions [9] allows one to write $W(z)$ as

$$W(z) = \sum_{i=1}^{\mathcal{N}} \left(\frac{\mathcal{A}_i}{(z+\rho_i)} + \frac{\mathcal{B}_i}{(z+\rho_i)^2} + \frac{\mathcal{C}_i}{(z+\rho_i)^3} \right), \tag{1.14}$$

where $\rho_i = m_i^2/\mu_{\mathrm{had}}^2$, with m_i the mass of the resonance i; and $\mathcal{A}_i, \mathcal{B}_i, \mathcal{C}_i$ are constants. In the large-N_c limit, the sum is in principle extended to an infinite number of resonances,

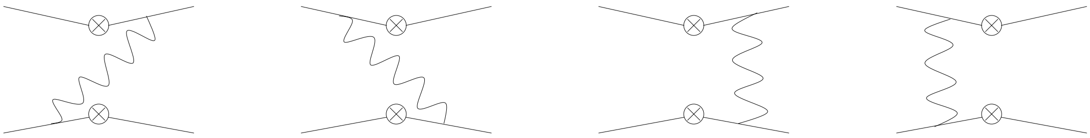

Figure 2: Diagrams contributing to the dimension 6 terms in the OPE in eq. (2.1).

i.e. $\mathcal{N} \to \infty$. On the other hand, as mentioned above, the approximation $W_{HA}(z)$ is defined by restricting the sum in eq. (1.14) to a finite $\mathcal{N}$. In particular, the work of ref. [2] restricted $\mathcal{N}$ to just one vector resonance, whose residues where matched onto the leading and next-to-leading terms in the chiral expansion but only to the leading (nontrivial) term in the operator product expansion of $W(z)$. This way it was obtained that $\hat{B}_K = 0.38 \pm 0.11$; with the error being an estimate of uncalculated terms, including $1/N_c$ corrections. The purpose of the present work is to compute the next-to-leading term in the operator product expansion of $W(z)$ at large-z, and its impact on the previous value for $\hat{B}_K$.

2. Dimension 8 operators in the weak OPE

Let us consider the Green's function (1.6). The large-Q^2 behavior of this function is governed by the Operator Product Expansion

$$\int d^4x\ e^{iqx}\ T\left\{L_\mu^{\bar{s}d}(x) L^\mu_{\bar{s}d}(0)\right\} = \sum_i c_i^{(6)}(Q^2)\mathcal{O}_i^{(6)} + \sum_i c_i^{(8)}(Q^2)\mathcal{O}_i^{(8)} + \cdots\,, \qquad (2.1)$$

where the first (second) sum runs over a set of dimension six (eight) local operators to be discussed in the following. In the case of dimension-six operators this set is actually limited to just one, to wit

$$\mathcal{O}^{(6)} = \bar{s}_L\gamma^\mu d_L(0)\bar{s}_L\gamma_\mu d_L(0)\,, \qquad (2.2)$$

with the Wilson coefficient given, to lowest order in $\alpha_s = g_s^2/4\pi$, by

$$c^{(6)}(Q^2) = i\frac{12\pi\alpha_s}{Q^4}\,. \qquad (2.3)$$

The relevant diagrams are listed in figure 2.

The determination of the dimension-eight operators is more involved. We used the Schwinger's operator method and obtained (after use of the equations of motion)

$$\begin{aligned}
\mathcal{O}_1^{(8)} &= \bar{s}\overleftarrow{\mathcal{D}_\mu}\overleftarrow{\mathcal{D}^\mu}\Gamma_\nu^a d(0)\ \bar{s}\Gamma_a^\nu d(0) + \bar{s}\Gamma_\nu^a\mathcal{D}_\mu\mathcal{D}^\mu d(0)\ \bar{s}\Gamma_a^\nu d(0) + \bar{s}\Gamma_a^\nu d(0)\ \bar{s}\overleftarrow{\mathcal{D}_\mu}\overleftarrow{\mathcal{D}_\mu}\Gamma_\nu^a d(0) + \\
&\quad + \bar{s}\Gamma_a^\nu d(0)\ \bar{s}\Gamma_\nu^a\mathcal{D}_\mu\mathcal{D}^\mu d(0)\,, \\
\mathcal{O}_2^{(8)} &= \bar{s}\Gamma_a^\nu\mathcal{D}_\mu d(0)\ \bar{s}\Gamma_\nu^a\mathcal{D}^\mu d(0) + \bar{s}\overleftarrow{\mathcal{D}_\mu}\Gamma_\nu^a d(0)\ \bar{s}\overleftarrow{\mathcal{D}^\mu}\Gamma_a^\nu d(0)\,, \\
\mathcal{O}_3^{(8)} &= \bar{s}\overleftarrow{\mathcal{D}_\mu}\Gamma_a^\nu d(0)\ \bar{s}\Gamma_\nu^a\mathcal{D}^\mu d(0) + \bar{s}\Gamma_\nu^a\mathcal{D}_\mu d(0)\ \bar{s}\overleftarrow{\mathcal{D}^\mu}\Gamma_a^\nu d(0)\,, \\
\mathcal{O}_4^{(8)} &= \bar{s}\overleftarrow{\mathcal{D}_\mu}\Gamma_a^\nu d(0)\ \bar{s}\overleftarrow{\mathcal{D}_\nu}\Gamma_a^\mu d(0) + \bar{s}\Gamma_\nu^a\mathcal{D}_\mu d(0)\ \bar{s}\Gamma_a^\mu\mathcal{D}^\nu d(0)\,, \\
\mathcal{O}_5^{(8)} &= \bar{s}\overleftarrow{\mathcal{D}_\mu}\Gamma_a^\nu d(0)\ \bar{s}\Gamma_a^\mu\mathcal{D}_\nu d(0) + \bar{s}\Gamma_\nu^a\mathcal{D}_\mu d(0)\ \bar{s}\overleftarrow{\mathcal{D}^\nu}\Gamma_a^\mu d(0) \\
\mathcal{O}_6^{(8)} &= g_s\ \tilde{G}_{\mu\nu}^a(0)\ \left\{\bar{s}\Gamma_a^\mu d(0)\ \bar{s}\Gamma^\nu d(0) - \bar{s}\Gamma^\mu d(0)\ \bar{s}\Gamma_a^\nu d(0)\right\}\,,
\end{aligned} \qquad (2.4)$$

where the following conventions were adopted:

$$\Gamma_a^\mu = \frac{\lambda_a}{2}\,\gamma^\mu\,\frac{1-\gamma_5}{2}\,, \qquad \mathcal{D}_\mu = \partial_\mu - ig_s A_\mu\,, \qquad A_\mu = \frac{A_\mu^a \lambda_a}{2}$$
$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_s[A_\mu, A_\nu]\,, \qquad \tilde{G}_{\mu\nu} = \frac{1}{2} G^{\rho\sigma}\epsilon_{\rho\sigma\mu\nu}\,, \quad \text{with} \quad \epsilon^{0123} = +1\,. \tag{2.5}$$

The corresponding Wilson coefficients are

$$c_i^{(8)} = i\frac{4\pi\alpha_s}{Q^6}\;\eta_i^{(8)}\,, \tag{2.6}$$

where

$$\eta_1^{(8)} = \frac{5}{3}\,, \qquad \eta_2^{(8)} = \frac{22}{3}\,, \qquad \eta_3^{(8)} = \frac{8}{3}$$
$$\eta_4^{(8)} = \frac{18}{3}\,, \qquad \eta_5^{(8)} = \frac{16}{3}\,, \qquad \eta_6^{(8)} = \frac{1}{N_c}\,. \tag{2.7}$$

This result agrees with the calculation in ref. [6] if one takes into account that, in the present case, the specific flavor structure $(\bar{s}d)^2$ leads to some simplifications. In particular, it allows one to rewrite $\mathcal{Q}_4$ in ref. [6] in terms of the $\mathcal{O}_{4,5}$ operators in eq. (2.4).

Inserting the operators (2.2), (2.4) in eqs. (2.1) and (1.6)–(1.11) one can compute the expansion in powers of $1/z$ of the function $W(z)$ in eq. (1.11). The terms of $\mathcal{O}(z^{-1})$ were already computed in ref. [2]. In doing the calculation of the next-to-leading terms, of $\mathcal{O}(z^{-2})$, we notice that several further simplifications take place. Firstly, since the contribution is proportional to α_s — see eq. (2.3) and (2.6) — , the large-N_c limit allows one to use factorization. Secondly, the contribution from $\mathcal{O}_6$ is $1/N_c$ suppressed due to the value of its Wilson coefficient, eq. (2.7). Thirdly, one is only interested in the leading term in the $l \to 0$ limit of the Green's function (1.6). Since, after Fierzing, one ends up with matrix elements of the form

$$\langle 0|\bar{s}_L \Gamma_\nu \mathcal{D}_\mu d_L(0)|K(l)\rangle \sim a\; g_{\mu\nu} + b\; l_\mu l_\nu\,, \tag{2.8}$$

by contracting with $g^{\mu\nu}$ one immediately concludes that $a \sim \mathcal{O}(l^2)$ and, consequently, $\mathcal{O}_{2,3,4,5}$ can be neglected since they yield contributions to eq. (1.8) with an extra power of l^2. Therefore, the term of $\mathcal{O}(z^{-2})$ in the large-z expansion of $W(z)$ in eq. (1.11) is governed solely by $\mathcal{O}_1$. Furthermore, using $D^2 = \not{D}^2 + \frac{g_s}{2}\sigma_{\mu\nu}G^{\mu\nu}$ in the expression for $\mathcal{O}_1$, and the previous considerations, lead to the conclusion that the final result can be given in terms of a *single* matrix element,[6] namely

$$\langle 0|g_s \bar{s}_L \tilde{G}^a_{\mu\nu}\lambda_a\gamma^\mu d_L|K(l)\rangle = -i\sqrt{2}F_0\;\delta_K^2\;l_\nu\,, \tag{2.9}$$

where δ_K^2 is a parameter to be determined in the next section. In conclusion, the OPE of $W(z)$ in eq. (1.11) can be written in terms of this parameter δ_K^2 as

$$W^{\rm OPE}(z) = \frac{24\pi\alpha_s F_0^2}{\mu_{\rm had}^2}\;\frac{1}{z}\left[1 + \frac{\epsilon}{12}(5+\kappa) + \frac{10}{9}\frac{\delta_K^2}{\mu_{\rm had}^2}\;\frac{1}{z}\right] + \mathcal{O}\left(\frac{1}{z^3}\right). \tag{2.10}$$

[6]This is related to the fact that $\bar{s}\gamma^\mu\widetilde{G}_{\mu\nu}d$ is the only nontrivial dimension-five operator in the chiral limit with these quantum numbers [10].

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Since the term proportional to δ_K^2 yields an ultraviolet convergent integral in eq. (1.11), it can be computed in 4 dimensions. This is why there is no ϵ dependence in eq. (2.10) accompanying this term.

3. Determination of δ_K^2

The mixed quark-gluon matrix element (2.9) was determined in the literature in [10, 11]. Both approaches relied on an analysis of Borel Sum Rules applied to two-point functions with nonvanishing contribution from the perturbative continuum. Although their final result for δ_K^2 happened to be in agreement, there were some criticisms raised by [11] on the analysis of [10], in particular on the choice for the onset of this perturbative continuum, s_0. This partly motivated us to do a reanalysis of the matrix element (2.9).

A particularly simple way to bypass the need for choosing a value for s_0 consists in looking at Green's functions which are order parameters of spontaneous chiral symmetry breaking since in these functions the perturbative continuum vanishes by construction. This procedure has proved fruitful in several contexts [12] and will be applied to the present case as well.

Consider the Green's function

$$\widetilde{\Pi}^{LR}_{\mu\nu}(Q^2) = i \int d^4x \; e^{iqx} \langle 0 \,| T \left\{ \frac{g_s}{2} \bar{s}_L \tilde{G}_{\alpha\mu} \gamma^\alpha d_L(x) \; \bar{d}_R \gamma_\nu s_R(0) \right\} |0\rangle \,. \tag{3.1}$$

Lorentz invariance guarantees that, in the chiral limit, it has the following tensorial structure

$$\widetilde{\Pi}^{LR}_{\mu\nu}(q) = (q_\mu q_\nu - g_{\mu\nu} q^2)\; \widetilde{\Pi}^{LR}(q^2)\,. \tag{3.2}$$

The large-q^2 fall-off of the function (3.2) can be straightforwardly computed to be

$$\widetilde{\Pi}^{LR}(q^2) = -\frac{2\pi}{9} \frac{\alpha_s \langle \bar{\psi}\psi \rangle^2}{Q^4} \,, \qquad Q^2 \equiv -q^2 \,, \tag{3.3}$$

where factorization of the four-quark condensate in the large-N_c limit has been used. Since $\widetilde{\Pi}_{LR}$ obeys the unsubtracted dispersion relation

$$\widetilde{\Pi}^{LR}(Q^2) = \int_0^\infty dt \frac{1}{\pi} \frac{Im\, \widetilde{\Pi}^{LR}(t)}{t + Q^2} \,, \tag{3.4}$$

we can consider as the minimal hadronic approximation the following spectral function

$$\frac{1}{\pi} Im\, \widetilde{\Pi}^{LR}(t) = -\frac{F_0^2 \delta_K^2}{4} \delta(t) + \frac{f_V^2 \delta_V^2}{8} \delta(t - m_V^2) \tag{3.5}$$

in which one introduces, besides the Goldstone boson, a vector resonance. Inserting (3.4) into (3.5) and expanding for large Q^2 one obtains, upon comparison with the OPE of (3.3), the two Weinberg-like sum rules

$$\begin{aligned} \frac{F_0^2 \delta_K^2}{4} &= \frac{f_V^2 \delta_V^2}{8} \\ \frac{f_V^2 \delta_V^2}{8} m_V^2 &= \frac{2\pi}{9} \; \alpha_s \langle \bar{\psi}\psi \rangle^2 \,, \end{aligned} \tag{3.6}$$

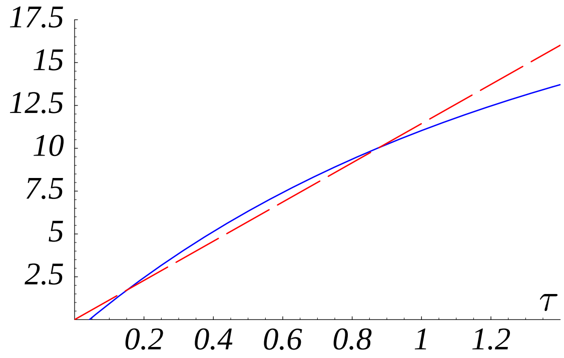

Figure 3: Plot of the right and lefthand side of eq. (3.11) (in arbitrary units), corresponding to the dashed and solid curve, respectively. τ is expressed in GeV^{-2}.

from which the unknown δ_V^2 and δ_K^2 parameters are readily determined to be

$$\begin{aligned} \delta_V^2 &= \frac{16\pi}{9}\,\frac{\alpha_s\langle\bar\psi\psi\rangle^2}{f_V^2 m_V^2}\,, \\ \delta_K^2 &= \frac{8\pi}{9}\,\frac{\alpha_s\langle\bar\psi\psi\rangle^2}{F_0^2 m_V^2}\,. \end{aligned} \tag{3.7}$$

Using $F_0 \simeq 0.087\,\mathrm{GeV}$, $f_V \simeq 0.15$, $m_V \simeq 0.77\,\mathrm{GeV}$, $\alpha_s(2\,\mathrm{GeV}) \simeq 1/3$ and $\langle\bar\psi\psi\rangle(2\,\mathrm{GeV}) = -(280 \pm 30\,\mathrm{MeV})^3$ one obtains[7]

$$\delta_K^2 = 0.12 \pm 0.07\,\mathrm{GeV}^2\,, \qquad \delta_V^2 = 0.06 \pm 0.04\,\mathrm{GeV}^4\,. \tag{3.8}$$

Strictly speaking, both parameters δ_K^2 and δ_V^2 depend on the renormalization scale μ. This dependence, however, is very small. As a matter of fact, according to ref. [14] the combination

$$\alpha_s(\mu)^{-4/11}\; g_s \bar s_L \gamma^\mu \tilde G_{\mu\nu} d_L \tag{3.9}$$

is renormalization group invariant. This means that the μ dependence of the parameters $\delta_{K,V}^2$ in eq. (3.7) yields a variation of 10% if μ is varied in the range $1 \le \mu \le 2\,\mathrm{GeV}$.

Apart from the dependence on the quark condensate, in order to check how much the result (3.8) depends on the two-particle decomposition of the spectral function in eq. (3.5), we can now repeat the analysis by introducing a third state. We take this state to be the first axial resonance. In this case, the spectral function reads

$$\frac{1}{\pi} Im\,\widetilde\Pi^{LR}(t) = -\frac{F_0^2\delta_K^2}{4}\;\delta(t) + \frac{f_V^2\delta_V^2}{8}\delta(t-m_V^2) + \frac{f_A^2\delta_A^2}{8}\delta(t-m_A^2)\,. \tag{3.10}$$

One can now apply the Borel transform to (3.10) and the OPE of (3.3) to get the relation

$$-\frac{F_0^2\delta_K^2}{4} + \frac{f_A^2\delta_A^2}{8}e^{-m_A^2\tau} + \frac{f_V^2\delta_V^2}{8}e^{-m_V^2\tau} = -\frac{2\pi}{9}\alpha_s\langle\bar\psi\psi\rangle^2\tau\,, \tag{3.11}$$

[7] We have added generous error bars in the quark condensate to include the present spread of values in this quantity[13]. This error is the dominant one.

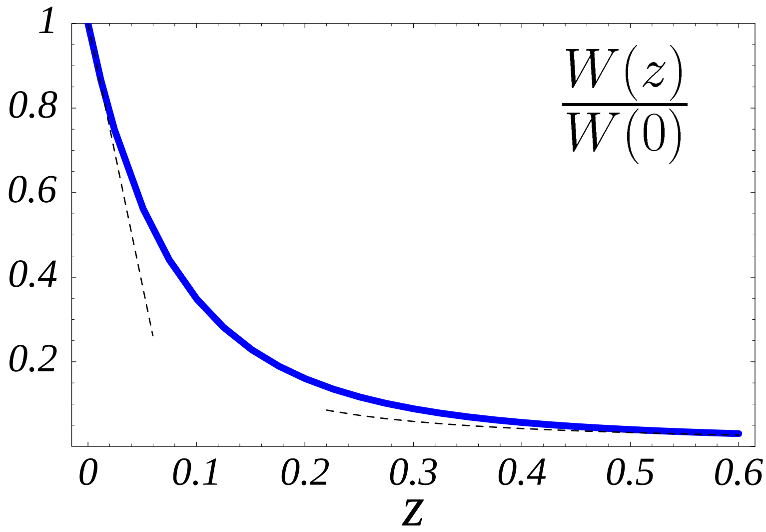

Figure 4: Plot of the large- and small-z expansions of the function $W(z)$ in eq. (1.11) as given by the OPE and Chiral Theory, respectively (dashed curves). The solid curve corresponds to the interpolating function $W_{HA}(z)$ obtained in eq. (4.1).

where τ is the Borel parameter. The equality above is fulfilled within a rather wide window of duality, namely $0 \lesssim \tau \lesssim 1\,\mathrm{GeV}^{-2}$, as can be seen in figure 3. Using $f_A = 0.08$ and $m_A = 1.2\,\mathrm{GeV}$, from this plot one extracts values for δ_K^2, δ_V^2 which agree with the ones obtained in (3.8). There is, therefore, consistency in the result obtained with the two methods. For the new parameter, δ_A^2, one obtains $\sim 0.05\,\mathrm{GeV}^4$. We remark that the value of δ_K^2 is a bit lower than the results obtained in refs. [10, 11]. To the best of our knowledge there is no lattice evaluation of δ_K^2.

4. Numerical evaluation of $\widehat{B}_K$ and conclusions

In ref. [2] the correction from dimension-eight operators was not considered and matching to the OPE was accomplished with just a single vector resonance whose mass was taken to be the physical ρ mass. In the present case we have a further constraint given by the new large-z behavior in eq. (2.10) caused by the presence of the parameter δ_K^2. Although one might think that this new constraint could allow one to determine the mass of the vector resonance, it turns out that this is actually not possible: the positive value of δ_K^2 would force the ρ mass to be purely imaginary. Therefore one must consider a further resonance.

Besides vectors, scalar particles also contribute to the coupling L_3[15, 16]. The dependence of the function $W(z)$ on L_3 at low-z found in [2] — see eq. (4.2) — suggests, therefore, a scalar particle as the new resonance in the sum (1.14). Furthermore, a scalar particle appears as a single pole in the function $W(z)$ in eq. (1.14) as a consequence of the quantum numbers being exchanged in the Green's function (1.6) (see figure 1). Interestingly, one scalar residue is all one needs to balance the new constraint from the OPE encoded in δ_K^2 through eq. (2.10). As to the scalar mass, which remains undetermined, we allow for the generous variation $m_s = 900 \pm 400\,\mathrm{MeV}$.

Gathering all the pieces, one has as the interpolating function

$$W_{HA}(z;\{S,V\}) = \frac{\mathcal{A}_V}{(z+\rho_V)} + \frac{\mathcal{B}_V}{(z+\rho_V)^2} + \frac{\mathcal{C}_V}{(z+\rho_V)^3} + \frac{\mathcal{A}_S}{(z+\rho_S)}\,, \tag{4.1}$$

to be matched onto

$$W^{\chi PT}(z) = 6 - 24\,\frac{\mu_{\rm had}^2}{F_0^2}\,\Big(2L_1 + 5L_2 + L_3 + L_9\Big)\,z + \mathcal{O}\left(z^2\right) \tag{4.2}$$

at low energies, and

$$W^{\rm OPE}(z) = \frac{24\pi\alpha_s F_0^2}{\mu_{\rm had}^2}\,\frac{1}{z}\,\left[1 + \frac{\epsilon}{12}(5+\kappa) + \frac{10}{9}\,\frac{\delta_K^2}{\mu_{\rm had}^2}\,\frac{1}{z}\right] + \mathcal{O}\left(\frac{1}{z^3}\right) \tag{4.3}$$

at high energies. This results in the following 4 constraints:

$$\begin{aligned}
\mathcal{A}_V + \mathcal{A}_S &= \frac{24\pi\alpha_s F_0^2}{\mu_{\rm had}^2}\left[1 + \frac{\epsilon}{12}(5+\kappa)\right] \\
\mathcal{B}_V - \mathcal{A}_V\rho_V - \mathcal{A}_S\rho_S &= \frac{24\pi\alpha_s F_0^2}{\mu_{\rm had}^4}\left(\frac{10}{9}\delta_K^2\right) \\
\frac{\mathcal{A}_V}{\rho_V} + \frac{\mathcal{B}_V}{\rho_V^2} + \frac{\mathcal{C}_V}{\rho_V^3} + \frac{\mathcal{A}_S}{\rho_S} &= 6 \\
\frac{\mathcal{A}_V}{\rho_V^2} + 2\frac{\mathcal{B}_V}{\rho_V^3} + 3\frac{\mathcal{C}_V}{\rho_V^4} + \frac{\mathcal{A}_S}{\rho_S^2} &= 24\frac{\mu_{\rm had}^2}{F_0^2}\left(2L_1 + 5L_2 + L_3 + L_9\right),
\end{aligned} \tag{4.4}$$

for the 4 unknowns $\mathcal{A}_S, \mathcal{A}_V, \mathcal{B}_V$ and $\mathcal{C}_V$. The combination $(2L_1 + 5L_2 + L_3 + L_9)$ in the last equation equals 11.2×10^{-3} if its experimental value evaluated at the ρ mass scale is used [17].[8] This combination is renormalization scale dependent, but this dependence can be neglected as it is subleading in the $1/N_c$ expansion. Numerically, it amounts to 30% if μ is varied in the range $0.5 \leq \mu \leq 1\,\mathrm{GeV}$. The coupling α_s is evaluated at the scale $\mu_{\rm had}$, which is chosen in the range $1 - 2\,\mathrm{GeV}$. All these variations will be included in the final error.

Figure 4 shows the low- and high-z behavior of the function $W(z)$ given by its chiral and operator product expansion (dashed curves) together with the interpolator $W_{HA}(z)$ in eq. (4.1) (solid curve).

The main effect from the dimension-eight operators is to push the large-z tail of the function $W(z)$ slightly upwards, as a consequence of δ_K^2 being positive. This implies a larger value of the integral in eq. (1.11), i.e. a larger area under the curve and, consequently, a smaller value for $\widehat{B}_K$. One can get a rough idea about the size of the contribution from the dimension-eight operators by noticing that the δ_K^2 in eq. (4.3) generates a correction to the coupling $g_{\Delta S=2}$ in eq. (1.11) given by

$$\begin{aligned}
\delta g_{\Delta S=2} &\sim -\,\frac{\mu_{\rm had}^2}{32\pi^2 F_0^2}\;\frac{24\pi\alpha_s(\mu_{\rm had})F_0^2}{\mu_{\rm had}^2}\;\frac{10}{9}\,\frac{\delta_K^2}{\mu_{\rm had}^2}\int_{\sim\frac{\mu_{\rm ope}^2}{\mu_{\rm had}^2}}^{\infty}\frac{dz}{z^2}\,, \\
&\sim -\,\frac{15}{18}\,\frac{\alpha_s(\mu_{\rm had})}{\pi}\,\frac{\delta_K^2}{\mu_{\rm ope}^2}\,,
\end{aligned} \tag{4.5}$$

[8] If the results of ref. [18, 19] are used, this combination is 8.7×10^{-3} (11.7×10^{-3}), respectively.

where $\mu_{\rm ope}$ is the scale above which the large-z expansion (i.e. the OPE) starts making sense. Numerically, one obtains $|\delta g| \lesssim 0.03$ when $\mu_{\rm ope} \sim 1\,{\rm GeV}$ and the value for δ_K^2 in eq. (3.8) are used.

To get a more accurate result one has to use the solution for $\mathcal{A}_S, \mathcal{A}_V, \mathcal{B}_V$ and $\mathcal{C}_V$ from eqs. (4.4) to construct the hadronic approximation W_{HA} in eq. (4.1). One can then explicitly perform the integral in (1.11) to obtain, after multiplication by the Wilson coefficient in eq. (1.2) according to eq. (1.13), a cancelation of scale and scheme dependence [2] with the following result[9]

$$\widehat{B}_K = \left(\frac{1}{\alpha_s(\mu_{\rm had})}\right)^{3/11} \frac{3}{4}\Bigg[1 - \frac{\alpha_s(\mu_{\rm had})}{\pi}\frac{1229}{1936} + \mathcal{O}\left(\frac{N_c\alpha_s^2(\mu_{\rm had})}{\pi^2}\right) - \\ - \frac{\mu_{\rm had}^2}{32\pi^2 F_0^2}\left(-\mathcal{A}_V \log\rho_V - \mathcal{A}_S\log\rho_S + \frac{\mathcal{B}_V}{\rho_V} + \frac{1}{2}\frac{\mathcal{C}_V}{\rho_V^2}\right)\Bigg]. \quad (4.6)$$

Using now the values $\alpha_s(2\,{\rm GeV}) \simeq 0.33$, $F_0 = 0.087\,{\rm GeV}$, $1\,{\rm GeV} \le \mu_{\rm had} \le 2\,{\rm GeV}$, the masses $m_V = 0.77 \pm 0.03\,{\rm GeV}$, $m_S = 0.9 \pm 0.4\,{\rm GeV}$ and the matrix element $\delta_K^2 = 0.12 \pm 0.07\,{\rm GeV}^2$, the expression (4.6) yields

$$\hat{B}_K = 0.36 \pm 0.15\,, \quad (4.7)$$

where the error quoted embraces an estimate of uncalculated $1/N_c$ corrections and the "noise" created by the uncertainty in all the parameters, taken one at a time.

We would like to conclude with a comparison with other results. Since the $\Delta S = 2$ operator (1.5) sits in the $(27_L, 1_R)$ representation of flavor ${\rm SU}(3)_L \times {\rm SU}(3)_R$, the result (4.7) translates into a value for the coupling constant g_{27} which governs $\Delta I = 3/2$ processes, such as $K^+ \to \pi^+\pi^0$, namely [20, 21]

$$g_{27} = \frac{4}{5}\hat{B}_K = 0.29 \pm 0.12\,. \quad (4.8)$$

This number is in very good agreement with a recent extraction [22] of g_{27} from $K \to 3\pi$ decays which yields, after subtraction of chiral corrections (within reasonable assumptions), the value $g_{27} \simeq 0.24$.

However, because chiral symmetry is much more difficult to have on the lattice than on the continuum [23], the situation concerning the value of $\hat{B}_K$ in numerical simulations in the chiral limit is not totally clear. It is seen in lattice data that $\hat{B}_K$ dips as quark masses go to zero [24], but the errors do not yet allow a sufficiently accurate extraction.[10] Nevertheless, we remark that two lattice collaborations have recently obtained $\hat{B}_K \simeq 0.3 - 0.4$ when extrapolated to the chiral limit [25].

Finally, alhough $\hat{B}_K$ in the chiral limit may be an interesting problem per se, it is clear that Nature is not in this limit and, therefore, that it is of the utmost importance to compute chiral corrections to the result (4.7). As a matter of fact, lattice results [27] and

[9]The quadratic dependence on μ_{had} is actually fictitious as is canceled by that of $\mathcal{A}_{V,S}$, $\mathcal{B}_V$ and $\mathcal{C}_V$, see eq. (4.4).

[10]See ref. [26] for a new strategy to try to overcome the difficulties.

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phenomenological analysis [28] seem to favor a large value for $\hat{B}_K$ at the physical kaon mass of the order of twice the value in (4.7). This would imply that the $\mathcal{O}(m_q/N_c)$ corrections to our result (4.7) are very large; a result which should be understood by analytic methods. That this is not impossible was shown in the calculation of ref. [29]. Even though within a model with notable differences with respect to QCD [30], the authors of ref. [29] obtained a value of $\hat{B}_K$ in the chiral limit in agreement with our result (4.7) while at the same time producing a larger $\hat{B}_K$ at the physical kaon mass, in agreement with refs. [27, 28].

Acknowledgments

We would like to thank M. Knecht and E. de Rafael for discussions and a careful reading of the manuscript. We would also like to thank E. de Rafael for pointing out the convenience of the Green's function (3.1) in the determination of δ_K^2. We would also like to thank the warm hospitality received at the CPT at Marseille. Work supported by CICYT-FEDER-FPA2002-00748, 2001-SGR00188, TMR EC-Contract No. HPRN-CT-2002-00311 (EURIDICE) and the France-Spain accion integrada "Picasso".

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